

Adaptive Pricing without Explicit Knowledge of Users Traffic Demands and Utility Functions

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Abstract—The problem of pricing for a telecommunication network is investigated with respect to the users' sensitivity to the pricing structure. A functional optimization problem is formulated, in order to compute price reallocations as functions of data collected in real time during the network evolution. No a-priori knowledge about the users' utility functions and the traffic demands is required, since adaptive reactions to the network conditions are sought in real time. To this aim, a neural approximation technique is studied to exploit an optimal pricing control law, able to counteract traffic changes with a small on-line computational effort.

Keywords—Network Pricing, User Sensitivity, Functional Optimization, Neural Control

I. INTRODUCTION

NETWORK pricing is an issue widely treated in the literature. In the last decade, a few models have been proposed to address the network management through the pricing structure [1]. In this paper we pursue the calculation of an optimal pricing control law as a function of the *Guaranteed Performance* (GP) and *Best Effort* (BE) users' responsiveness to the pricing structure, by exploiting data collected in real time during the system's evolution. In the literature, GP and BE pricing mechanisms are usually analyzed separately. We shall start with an insight into the pricing models related to the two traffic types, highlighting their advantages and drawbacks. Then, we formulate our pricing scheme to face the envisaged problems.

A. Utility-based pricing and Best Effort services

The concept of *utility function* has been introduced in the telecommunication literature to depict the *Quality of Service* (QoS) as appreciated by the users. It is possible to define the utility of a user r as a function of the user's bandwidth assignment x_r , namely $U_r(x_r)$. Such function describes how sensitive user r is to changes in x_r . In the context of pricing, it is useful to think of it as the amount of money user r is willing to pay for a certain x_r . Let a telecommunication network be composed by a set J of unidirectional links and a set R of users (*source-destination* (SD) pairs). Link j has capacity c_j , $J(r)$ is the subset of J containing the links traversed by user r , $R(j)$ is the subset of users traversing link j . Let

$A = \{A_{jr}, j \in J, r \in R\}$ be the matrix assigning resources to users ($A_{jr} = 1$ if link j is traversed by user r 's traffic, $A_{jr} = 0$, otherwise). In such a context, formulated in [2], each user accessing the network maximizes his/her utility with respect to the assigned price p^r , i.e., the bandwidth demand x_r is ruled by (1) below (m_r and M_r denote the lower and upper limits of the bandwidth domain, respectively).

$$\max_{x_r \in [m_r, M_r]} (U_r(x_r) - x_r p^r) \quad (1); \quad p^r = \sum_{j \in J(r)} p_j \quad (2)$$

If we interpret p_j as the price per unit bandwidth at link j , then p^r is the total price per unit bandwidth for all links in the path of user r . We now briefly recall the results of [2, 3] to underline the fact that a decentralized implementation of a congestion-dependent pricing is available to maximize the so-called *social welfare*, i.e.:

$$\max_{x_r \in [m_r, M_r]} \sum_r U_r(x_r); \quad \text{subject to } A \cdot x \leq c; x \geq 0 \quad (3)$$

when all users react to prices as outlined in (1). A , x , c are the aggregate vectors of the assigned resources, users' rates and link capacities, respectively. In brief, the key idea is to exploit the *Lagrangian* decomposition of (3), thus giving rise to a flow control mechanism of the form [3]:

$$x_r(p^r(t)) = [U_r^{-1}(p^r(t))]_{m_r}^{M_r}; \quad (4)$$

$$p_j(t+1) = p_j(t) - \eta \cdot \left[c_j - \sum_{r \in R(j)} x_r(p^r(t)) \right] \quad (5)$$

$x_r(p^r(t))$ represents the solution of (1), $[z]_a^b = \min\{\max\{z, a\}, b\}$, U_r^{-1} denotes the inverse of the utility function derivative and η is the gradient step-size. At each iteration, user r individually solves (1) through (4) and sets the rate on the respective SD path $J(r)$ to $x_r(p^r(t))$. Each link $j \in J(r)$ then updates its price p_j according to (5), it communicates the new prices to users $r \in R(j)$, whose transmission rate must be changed according to (4), and the cycle repeats.

This mechanism is appropriate for contracts with flexible guarantees, related to “elastic” applications. A drawback of utility-based pricing mechanisms is related to the *Service Provider’s* (SP) perception of utility functions. Even if some works investigate the user responsiveness to the perceived QoS and the tariff structure (see e.g., [4]), the notion of utility is actually difficult to measure or estimate. Real time traffic complicates the situation even more, since it requires QoS guarantees, and gives rise to a pricing structure involving the corresponding *effective bandwidth* of the services [1].

B. Traffic demands and Guaranteed Performance pricing

In practice, another approach for the optimization of network pricing is possible. The user’s responsiveness to the tariff structure can be related to the interarrival of the connection requests, disregarding any utility-based consideration. For each class of service, in which QoS requirements are guaranteed on an equivalent bandwidth fashion, [5-8] define frequency functions of the service requests $\lambda(\cdot)$ with respect to the assigned prices, namely: $\lambda(\cdot) = \lambda(p)$. In [9], a somehow similar approach is proposed, where $\lambda(p)$ is the packet arrival rate of BE traffic as a function of the price p . In the presence of multiple (K) service classes, let $\lambda(p)$ be the aggregate vector of the traffic laws $\lambda_{\kappa}(p_{\kappa})$; $\kappa=1, \dots, K$. Also in this case, the maximization of the network performance still remains an open issue, since different choices on the prices give rise to different evolutions of the system. It is possible to exploit proper mathematical instruments for the planning of the telecommunication network, but some severe drawbacks still remain unsolved. **(i)** The first one is related to the computational burden involved in such mathematical tools (*Dynamic Programming* in [5-7, 9], *Discrete Mathematical Programming* in [8]), which limits their application if real time reactions are needed. **(ii)** The second (and most important) one regards the assumption made on the perfect knowledge of the traffic laws $\lambda(p)$. Some knowledge on the user’s responsiveness to the pricing structure is supposed to be always in effect in [5-9]. If a perfect knowledge of users’ utility functions is difficult to assure in a real context, the same holds true for the estimate of the functions $\lambda(p)$ [6, 7]. **(iii)** Moreover, as underlined in [7, 8], it is worth noting that the optimal price allocation p^o can be only obtained through a centralized management of the network and, finally, **(iv)** the effect of time-varying bandwidth allocations (typical of BE services) is not taken into account if only the traffic laws $\lambda(p)$ of the GP users are considered.

C. The present approach

For these reasons, the study of a novel pricing mechanism, able to face the aforementioned drawbacks, reveals to be an attractive research topic. In this perspective, the idea of this work is to formulate a novel pricing control algorithm, such that: **(i)** it infers the optimal prices as functions of measures obtained in real time, without any on-line knowledge of the functions $\lambda(p)$ and $U(\cdot)$ ($U(\cdot)$ being the aggregate vector of the utility functions); **(ii)** it reacts on line to non-stationary

$\lambda(t, p)$ and $U(t, \cdot)$; **(iii)** it manages both GP and BE traffic, multiplexed together and sharing the available resources; **(iv)** it avoids any on-line computational burden; and **(v)** it is suitable for a decentralized control of the network.

The remainder of the paper is organized as follows. In the next Section we define both the network model and the revenue maximization problem. In Section III, we formulate our functional optimization approach and, in Section IV, a neural approximating technique is investigated to solve the problem. In Section V we validate our methodology through simulations and, in Section VI, we outline conclusions and future work.

II. NETWORK MODEL AND REVENUE OPTIMIZATION

A. The Guaranteed Performance case

For the time being, we consider the GP traffic only. With a notation that slightly differs from [8], let us consider H traffic routes within a telecommunication network. A route $h \in \{1, \dots, H\}$ is defined as a network path assigned to a group of GP users, according to the required source-destination nodes and with respect to the chosen routing plan. For each route, K different QoS treatments are available. A *service class* is identified in terms of assigned route, QoS treatment and assigned price $p_{h\kappa}$. For instance, in the MPLS terminology, a service class is equivalent to the concept of *Forwarding Equivalent Class* (FEC), established on a specific *Virtual Path* (VP). The corresponding equivalent bandwidth requests are denoted with $y_{h\kappa}$ and the corresponding traffic laws with $\lambda_{h\kappa}(p_{h\kappa})$. For instance, p may be in terms of [€/Mbps per minute] and y in [Mbps]. Following [8], a *service separation* among the service classes is implemented in each network node. This means that a buffer is provided for each class and a scheduler is supposed to guarantee a proper bandwidth allocation among the classes. By exploiting the traffic laws $\lambda(p)$, different network behaviors are possible in terms of shared resources and corresponding network performance (blocking probability, revenue, welfare, and so on). Such performance metrics are manageable by the SP, by implementing a proper tariff structure $p(t) = \text{col}\{p_{h\kappa}(t); h=1, \dots, H; \kappa=1, \dots, K\}, t \in [0, +\infty]$.

Disregarding, for the time being, any utility-based consideration, the first pricing problem is formulated as follows. **Pricing Allocation Problem** (PAP): find the optimal tariff assignment $p^o(t), t \in [0, +\infty]$, in such a way that the long-term average SP’s revenue defined in (6) below is maximized:

$$p^o(t) = \arg \max_{p(t)} \max_{\omega} E L[p, \omega];$$

$$L[p, \omega] = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \sum_{h=1}^H \sum_{\kappa=1}^K n_{h\kappa}(\tau) \cdot p_{h\kappa}(\tau) \cdot y_{h\kappa}(\tau) d\tau \quad (6)$$

where $n_{h\kappa}(t)$ is the number of active users of service class $h\kappa$ at time t , and ω represent a sample path of all stochastic variables involved in the generation of

$\{n_{h\kappa}(t), h=1, \dots, H, \kappa=1, \dots, K, t \in [0, +\infty]\}$. $n_{h\kappa}(t)$ depends on the chosen tariff structure $\mathbf{p}(\tau)$ (and corresponding arrival rates $\lambda(\tau), \tau \in [0, t]$) and on how the service classes have shared the available bandwidth under the assigned routing paths in the time interval $[0, t]$. For each new GP request, a *Call Admission Control* (CAC) agent is supposed to guarantee the network constraints due to the limited bandwidth of the links, thus rejecting the incoming call if sufficient resources are not available. We include in ω all the stochastic variables involved in the system, i.e., the aggregate vectors of the arrival process of connection requests (\mathbf{a}), of the call durations (\mathbf{d}) and of the bandwidth requirements (\mathbf{y}), which can follow complex interactions among the traffic sources through statistical multiplexing, i.e., $\omega = [\mathbf{a}, \mathbf{d}, \mathbf{y}]$.

III. A FUNCTIONAL OPTIMIZATION APPROACH

To face the above-described problems, we develop a functional optimization approach, by defining pricing reallocations as functions of an “*information vector*”, which summarizes the “*recent history*” of the network. Several “*inspector agents*” are supposed to be disseminated in the network and to perform the necessary measurements. More specifically, an inspector agent is assigned to each service class and monitors its temporal behaviour in real time. Let $\mathbf{m}_{h\kappa}(\cdot)$ be the information vector available for the $h\kappa$ inspector agent with respect to the state of the overall network. Different measurable variables may be grouped in the $\mathbf{m}_{h\kappa}(\cdot)$ vector, depending on the PAP we deal with. For instance, the variables of interest for PAP I are the number of GP requests received, for each service class, in the last time intervals of observation.

A distributed structure of *Decision Makers* (DMs) is responsible for the prices’ assignment. A DM is assigned to each network node where users submit connection requests. It computes new prices’ reallocations for the service classes crossing the node. We denote by \hat{t} a reasonable upper bound for the time delay necessary for the DM to obtain stable information updates about the $\mathbf{m}_{h\kappa}(\cdot)$ vectors from the inspector agents. Pricing reallocations are then performed by the DMs for each service class $h\kappa$ at consecutive time instants $t = k\hat{t}$, $k = 0, 1, \dots$, on the basis of a “*knowledge*” collected as:

$$\mathbf{I}_{h\kappa}(k\hat{t}) = \text{col}\{\mathbf{m}_{h\kappa}((k - \Xi)\hat{t}), \dots, \mathbf{m}_{h\kappa}((k - 1)\hat{t})\} \quad (8)$$

where Ξ denotes the depth of such finite horizon memory.

Let us consider the PAP I stated in (6). Let $J_{k\hat{t}}(\mathbf{p}(k\hat{t})) = E L_{k\hat{t}}[\mathbf{p}(k\hat{t}), \omega]$ be the average-reward, infinite-horizon functional cost after the price reallocation at time $k\hat{t}$:

$$L_{k\hat{t}}[\mathbf{p}(k\hat{t}), \omega] = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{k\hat{t}}^{k\hat{t}+T} \sum_{h=1}^H \sum_{\kappa=1}^K n_{h\kappa}(\tau) \cdot p_{h\kappa}(k\hat{t}) \cdot y_{h\kappa}(\tau) d\tau \quad (9)$$

and let $f_{h\kappa}(\mathbf{I}_{h\kappa}(k\hat{t}))$ be the “*price reallocation law for service class $h\kappa$* ” (i.e., $p_{h\kappa}(k\hat{t}) = f_{h\kappa}(\mathbf{I}_{h\kappa}(k\hat{t}))$). We must note that the decision function of the DM assigned to a specific node is composed by all the price reallocation laws related to the service classes crossing that node. For this reason, we denote by $\mathbf{f}(\cdot)$ and $\mathbf{I}(\cdot)$ the DM’s decision function and the related information vector, obtained by the composition of the information vectors (8) for the all service classes $h\kappa$ crossing the node. The revenue functional (9) becomes the basis of the following functional optimization problem. **Problem F-PAP** (*Functional - Pricing Allocation Problem*): find the optimal pricing reallocation function $\mathbf{f}^*(\cdot)$, such that the following functional performance index is maximized:

$$J_{k\hat{t}}[\mathbf{f}(\mathbf{I}(k\hat{t}))] = E L_{k\hat{t}}[\mathbf{f}(\mathbf{I}(k\hat{t})), \omega] \quad (10)$$

IV. THE OPTIMIZATION METHODOLOGY

In order to approximate the optimal pricing control law $\mathbf{f}^*(\cdot)$, we develop a modified version of the *Extended Ritz* method [10]. The Extended Ritz method is a technique suitable for the approximation of the solution of functional optimization problems, by fixing the structure of the decision functions. Such decision functions are constrained to take on the structure of approximating networks, i.e., linear combinations of (non linear) basis functions, containing free parameters to be optimized:

$$\begin{aligned} \bar{\mathbf{f}}(\mathbf{I}, \mathbf{w}) &= \text{col}\left\{\sum_{l=1}^v c_l^i \zeta(\mathbf{I}, \tilde{\mathbf{w}}_l) + c_0^i; i=1, \dots, H \cdot K\right\}; \\ \mathbf{w} &= \{\tilde{\mathbf{w}}_l, l=1, \dots, v, c_l^i, l=1, \dots, v, c_0^i, i=1, \dots, H \cdot K\} \end{aligned} \quad (11)$$

where $\zeta(\cdot)$ and v represent a suitable basis function and the number of basis functions used to build the approximator $\bar{\mathbf{f}}(\cdot, \cdot)$, respectively. Among the possible choices of structures of the form (11), we choose *one hidden layer feedforward neural networks* (due to their powerful approximation properties to face the possible exponential growth in the number of free parameters, needed to obtain an increasing degree of accuracy). We suppose that each price $p_{h\kappa}$ is constrained to a given domain, i.e. $p_{h\kappa} \in [p^{m_{h\kappa}}; p^{M_{h\kappa}}]$. In order to guarantee the fulfilment of such constraints, we compose the output of the neural network with a *normalization operator* $\mathbf{n}(\cdot)$. We thus obtain prices’ reallocations $\mathbf{p}(k\hat{t})$ at any time $k\hat{t}$ as:

$$\begin{aligned} \mathbf{p}(k\hat{t}) &= \mathbf{n}[\bar{\mathbf{f}}(\mathbf{I}(k\hat{t}), \mathbf{w})]; n_{h\kappa}(\bar{\theta}_{h\kappa}) = p^{m_{h\kappa}} + (p^{M_{h\kappa}} - p^{m_{h\kappa}}) \cdot \bar{\theta}_{h\kappa}; \\ \bar{\theta}_{h\kappa} &= \bar{f}_{h\kappa}(\mathbf{I}(k\hat{t}), \mathbf{w}), \bar{\theta}_{h\kappa} \in [0.0, 1.0], \forall h, \kappa \end{aligned} \quad (12)$$

We shall call “*neural pricing allocation function*” (NPAF) the aggregation of functions (11), obtained as composition of the neural networks and the normalization operators, and denote it by $\hat{\mathbf{f}}(\mathbf{I}(\cdot), \mathbf{w})$. It follows that a parametrized cost function is

obtained, by substituting the fixed structure of such NPAF into cost (10), depending on the parameter vector \mathbf{w} , thus leading to the following mathematical programming problem. **Problem F-PAP w** : find the optimal parameter vector \mathbf{w}^* , such that the cost function (15) is maximized:

$$E_{\omega} L_{ki} \left[\hat{\mathbf{f}}(\mathbf{I}(k\hat{t}), \mathbf{w}), \omega \right] = E_{\omega} \tilde{L}_{ki}(\mathbf{w}, \omega) \quad (13)$$

In this way, the F-PAP (11) has been reduced to an unconstrained nonlinear programming problem.

To solve such nonlinear programming problem, we should apply a gradient-based algorithm:

$$\mathbf{w}^{\chi+1} = \mathbf{w}^{\chi} - \xi \nabla_{\mathbf{w}} E_{\omega} \tilde{L}_{ki}(\mathbf{w}^{\chi}, \omega), \quad \chi = 0, 1, 2, \dots \quad (14)$$

where ξ is a fixed stepsize. However, the explicit computation of the expected cost and its gradient is a very hard task, even if closed-form formulas for the functional cost $\tilde{L}_{ki}(\cdot)$ were available [10]. We choose to compute a realization $\nabla_{\mathbf{w}} \tilde{L}_{ki}(\mathbf{w}^{\chi}, \omega^{\chi})$, instead of the gradient $\nabla_{\mathbf{w}} E_{\omega} \tilde{L}_{ki}(\mathbf{w}^{\chi}, \omega)$, and we apply the updating algorithm:

$$\mathbf{w}^{\chi+1} = \mathbf{w}^{\chi} - \xi_{\chi} \nabla_{\mathbf{w}} \tilde{L}_{ki}(\mathbf{w}^{\chi}, \omega^{\chi}), \quad \chi = 0, 1, 2, \dots \quad (15)$$

where the index χ denotes both the steps of the iterative procedure and the generation of the χ -th realization of the stochastic processes involved in ω . The components of the gradient $\nabla_{\mathbf{w}} \tilde{L}_{ki}(\mathbf{w}^{\chi}, \omega^{\chi})$ can be obtained by using the classical backpropagation equations for the training of neural networks. The backpropagation procedure must be initialized by means of the quantities $\frac{\partial L}{\partial p_{h\kappa}}$, $h = 1, \dots, H; \kappa = 1, \dots, K$ (i.e.,

the gradient $\nabla_{\mathbf{p}} L(\mathbf{p}, \omega^{\chi})$). Unfortunately, in our case, such quantities cannot be obtained analytically as in [10], because no closed-form of the functional cost $L(\cdot)$ is available. Hence, during the training phase (15), gradient estimates are computed as:

$$\frac{\partial L_{ki}(\mathbf{p}(k\hat{t}), \omega^{\chi})}{\partial p_{h\kappa}(k\hat{t})} \cong \frac{L_{ki}(\mathbf{p}_{h\kappa+}(k\hat{t}), \omega^{\chi}) - L_{ki}(\mathbf{p}(k\hat{t}), \omega^{\chi})}{\Delta_p} \quad (16)$$

where $\mathbf{p}_{h\kappa+}(k\hat{t})$ means that a “small” increase is carried out for $p_{h\kappa}(k\hat{t})$ (the $h\kappa$ -th component of the price vector $\mathbf{p}(k\hat{t})$), i.e., $p_{h\kappa+}(k\hat{t}) = p(k\hat{t}) + \Delta_p$.

V. PERFORMANCE EVALUATION AND DISCUSSION

A. Convergence behaviour of the control algorithm

A trivial optimal pricing assignment (easily computable through simulation comparisons) has to be reached for a small network (composed by one link and two service classes). As outlined above, a service class is defined as a routing path combined with a specific QoS treatment (in terms of equivalent

bandwidth assignment). The simulation scenario is built as follows. Two routes, whose traffic demands are $\lambda_i(p_i) = 1.0 \cdot p_i^{-1.05}$, $p_i \in [1; 100]$; $i = 1, 2$, belong to a single link of 10.0 Mbps capacity. The corresponding interarrival times are exponentially distributed. The aforementioned traffic laws are taken from [11] and reproduce the demand elasticity of a voice service. The mean duration of the calls is fixed to 10.0 minutes and is log-normally distributed. A neural network with 15 hyperbolic tangent neural units in the hidden layer and with a sigmoidal output layer has been used. The information vector $\mathbf{I}(\cdot)$ of the NPAF is composed by the numbers of GP requests received, for each class of service, in the previous 5 time intervals of observation (together with the corresponding pricing assignments as in (8)). A new interval of observation starts every hour and, as a consequence, $\Xi \hat{t}$ is fixed to 6 hour (the first hour of simulation is considered a transient period to meet the regime of the stochastic processes). The overall simulation time T in (6) after the price reallocation is set to 15 hours for each training step. The gradient stepsize ξ_{χ} of (15) is

taken as $\xi_{\chi} = \frac{1.0}{2.0 \cdot 10^5 + \chi}$ (thus assuring a decreasing behaviour as convergence requirement) and we have also

added a “momentum” term $\rho \cdot (\mathbf{w}^{\chi} - \mathbf{w}^{\chi-1})$, $\rho = 0.3$, to (15), as usually done in training neural networks to speed up convergence. The Δ_p parameter of the gradient estimation procedure (16) was fixed to 6.0. Figs. 1 and 2 show the revenue performance and the prices’ assignments during the training phase. For the simple network scenario under investigation, the lowest price values $p_i^o = 1.0$; $i = 1, 2$ (and the corresponding frequency of the interarrival requests $\lambda_i^o = \lambda_i(p_i^o)$) guarantee the best performance.



Figure 1. Simulation scenario I, revenue during training.

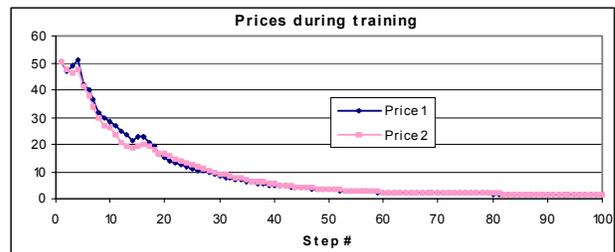
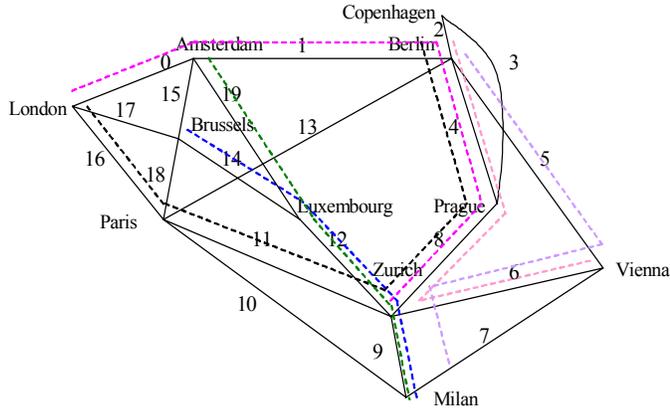


Figure 2. Simulation scenario I, prices during training.

B. Optimal pricing assignment in a real network scenario

The second simulation scenario makes use of the ‘COST 239’ experimental network topology under the voice service classes described above and deployed along the routes depicted in Fig. 3.

The capacity of the network links is fixed to 30.0 Mbps. The employed neural network structure is the same as the one of the previous simulation scenario.



Route 1: {0, 1, 4, 8}; --- Route 2: {4, 8, 6}; --- Route 3: {9, 12, 19}; ---
Route 4: {4, 8, 11, 16}; --- Route 5: {9, 12, 14}; --- Route 6: {5, 6, 9}; ---

Figure 3. Simulation scenario II, Topology of the test network.

Both the prices’ domain, the training parameters, the simulation horizon T in (6) and the Δ_p parameter were accurately tuned in order to speed up the convergence of the training phase: $p_i \in [1, \dots, 10], \forall i, \xi_\chi = \frac{1.0}{6.0 \cdot 10^4 + \chi}, \rho = 0.3, T = 1000$ hours, $\Delta_p = 1$.

The training procedure converges to the following pricing assignment:

$$\begin{aligned} p_1^o &= 5.35; p_2^o = 5.65; p_3^o = 3.62; \\ p_4^o &= 3.25; p_5^o = 1.79; p_6^o = 2.21. \end{aligned} \quad (17)$$

We report in Table 1 the most significant samples obtained over the performance index, just to highlight how solution (17) guarantees the best bandwidth sharing among the users, corresponding to an overall blocking probability of 0.9606%, which reveals to be the optimal one (in terms of revenue), as compared to the other blocking probability values (obtained in correspondence with different price allocations).

Pricing Assignment	Revenue	Blocking Probability
$p_i = 100.0, i=1, \dots, 6$	4.406673e+006	0.0
$p_i = 25.0, i=1, \dots, 6$	4.647619e+006	0.0
$p_i = 10.0, i=1, \dots, 6$	4.863280e+006	0.0
$p_i = 5.0, i=1, \dots, 6$	5.054878e+006	0.001132
(17)	5.125e+006	0.009606
$p_i = 4.0, i=1, \dots, 6$	5.087591e+006	0.006751
$p_i = 3.0, i=1, \dots, 6$	4.982476e+006	0.031672
$p_i = 2.0, i=1, \dots, 6$	4.426812e+006	0.136192
$p_i = 1.0, i=1, \dots, 6$	2.889035e+006	0.402790

Table 1. Simulation scenario II, Pricing assignments and performance.

VI. CONCLUSIONS AND FUTURE WORK

In this paper, a novel control mechanism has been studied to allow optimal price reallocations as a function of the state of the network.

Future work includes the application of further constraints (e.g., GP blocking probabilities) and analysis of the impact of decentralized versus centralized control using the NBAF.

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